

Suppressing Proton Decay by Cancellation in S_4 Flavor Symmetric Extra U(1) Model

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Abstract

We consider proton stability based on E_6 inspired extra U(1) model with S_4 flavor symmetry. In this model, a long life time of proton is realized by the flavor symmetry. One of the interesting effects of flavor symmetry is that the proton decay widths of $p \rightarrow \mu^+ X$ are suppressed by cancellation. This suppression mechanism is important in the case that Yukawa coupling constants are hierarchical. Our model predicts $p \rightarrow e^+ K^0$ has larger decay width than that of $p \rightarrow \mu^+ K^0$.

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1 Introduction

Supersymmetry is an elegant solution of hierarchy problem of Standard Model (SM) [1], however, a simple supersymmetric extension of SM suffers from non-conservation of baryon number and μ -problem. Therefore we must introduce new symmetry such as R-parity in MSSM, to suppress proton decay operators and μ -term. One of the solutions of μ -problem is given by introducing extra $U(1)$ gauge symmetry [2]. In this frame work, several new superfields such as singlet S , exotic quarks G, G^c , must be introduced to cancel gauge anomaly. This is the elegant solution of μ -problem, however proton instability is not solved because the baryon number violating interactions in superpotential of MSSM are replaced by single exotic quark interactions.

In the superpotential, there is no obvious distinction between baryon number violating trilinear terms and Yukawa interactions. Therefore it is natural to introduce flavor symmetry in suppressing baryon number violating operators. Especially non-Abelian discrete symmetry is good candidate for the flavor symmetry, because large mixing angles of Maki-Nakagawa-Sakata (MNS) mixing matrix may be explained simultaneously, and more simply, non-Abelian symmetry can be the reason why generation exists. At previous work, we explained $S_4 \times Z_2$ flavor symmetry not only realizes maximal mixing angle θ_{23} but also suppresses proton decay based on $SU(3)_c \times SU(2)_W \times U(1)_Y \times U(1)_X \times U(1)_Z$ gauge symmetry [3]. As the suppression mechanism of proton decay is complicated and model dependent, we give more detailed estimation of proton life time and investigate several flavon sectors in this paper.

In the several models of the non-Abelian discrete symmetries, although the direction of vacuum expectation values (VEVs) of flavons is crucial, the direction is selected by hand adding the explicit symmetry breaking terms. As such procedure is unnatural, we start from exactly flavor symmetric theory in the estimation of proton life time. The spontaneous flavor symmetry breaking realizes special VEV direction of flavons, which affects proton life time significantly. If the Yukawa hierarchy is realized by Froggatt-Nielsen mechanism, as $p \rightarrow e^+ X$ are suppressed due to the small coupling constants, $p \rightarrow \mu^+ X$ may dominate proton decay width. In our model, as $p \rightarrow \mu^+ X$ are suppressed by cancellation, $p \rightarrow e^+ K^0$ dominates the proton decay width.

This paper is organized as follows. In section 2, we estimate proton life time based on gauge non-singlet flavon model. In section 3, we modify the flavon sector by adding gauge-singlet flavon and Froggatt-Nielsen flavon. In section 4, we eliminate gauge non-singlet flavon and construct Dirac neutrino model. Finally we give conclusion of our analysis in section 5.

2 $S_4 \times Z_2$ flavor symmetric extra $U(1)$ model

At first we explain the basic structure of our model. We extend the gauge symmetry to $G_2 = SU(3)_c \times SU(2)_W \times U(1)_Y \times U(1)_X \times U(1)_Z$ which is the maximal subgroup of E_6 . In order to cancel gauge anomaly, we must add new superfields, such as SM singlet S , exotic quark G, G^c (hereafter we call them g-quark) and right handed neutrino (RHN) N^c . We can embed these superfields with MSSM superfields $Q, U^c, D^c, L, E^c, H^U, H^D$ into **27** of E_6 [4]. As the singlet S develops VEV and breaks $U(1)_X$ gauge symmetry, O (TeV) scale μ -term is induced naturally. In order to break $U(1)_Z$ and generate a large Majorana mass of RHN, we add SM singlet Φ, Φ^c . The gauge representations of superfields are given in Table 1 [3].

	Q	U^c	E^c	D^c	L	N^c	H^D	G^c	H^U	G	S	Φ	Φ^c
$SU(3)_c$	3	3^*	1	3^*	1	1	1	3^*	1	3	1	1	1
$SU(2)_W$	2	1	1	1	2	1	2	1	2	1	1	1	1
$y = 6Y$	1	-4	6	2	-3	0	-3	2	3	-2	0	0	0
x	1	1	1	2	2	0	-3	-3	-2	-2	5	0	0
z	-1	-1	-1	2	2	-4	-1	-1	2	2	-1	8	-8
R	-	-	-	-	-	-	+	+	+	+	+	+	+

Table 1: G_2 assignment of fields. Where the x, y and z are charges of $U(1)_X, U(1)_Y$ and $U(1)_Z$, and Y is hypercharge. R-parity $R = \exp \left[\frac{i\pi}{20} (3x - 8y + 15z) \right]$ is unbroken.

Under the gauge symmetry given in Table 1, the renormalizable superpotential is given by

$$W = Y^U H^U Q U^c + Y^D Q D^c H^D + Y^E H^D L E^c + Y^N H^U L N^c + Y^M \Phi N^c N^c + \lambda S H^U H^D + k S G G^c$$

$$+ M\Phi\Phi^c + Y^{QQG}GQQ + Y^{UDG}G^cU^cD^c + Y^{EUG}GE^cU^c + Y^{QLG}G^cLQ + Y^{NDG}GN^cD^c. \quad (1)$$

In this superpotential, unwanted terms are included in the second line. The first term of the second line is the mass term of singlets Φ, Φ^c which prevent singlets from developing VEVs. The other five terms of the second line are single g-quark interactions, which break baryon and lepton number and induce rapid proton decay. In the first line, we must take care of the flavor changing neutral currents (FCNCs) induced by extra Higgs bosons [5]. Therefore the superpotential Eq.(1) is not consistent at the present stage.

In order to stabilize proton, we introduce $S_4 \times Z_2$ flavor symmetry. If we assign G, G^c to S_4 triplet and quarks and leptons to doublet or singlet, the single g-quark interaction is forbidden. However, as the g-quark must never be stable from phenomenological reason, we assign Φ^c to S_4 triplet to break the flavor symmetry slightly. In this case, as Φ, Φ^c play the role of flavons, we call them gauge non-singlet flavons. In order to realize the large mixing angle of θ_{23} in the MNS matrix and suppress the Higgs-mediated FCNCs, we assign the superfields in our model as given in Table 2 [6].

In the non-renormalizable part of superpotential, the single g-quark interactions which contribute to the g-quark decay are given as follows

$$W \supset \frac{Y^{QQG}}{M_P^2}\Phi\Phi^cQQG + \frac{Y^{UDG}}{M_P^2}\Phi\Phi^cG^cU^cD^c + \frac{Y^{EUG}}{M_P^2}\Phi\Phi^cGE^cU^c + \frac{Y^{QLG}}{M_P^2}\Phi\Phi^cG^cLQ. \quad (2)$$

	Q_1	Q_2	Q_3	U_1^c	U_2^c	U_3^c	D_1^c	D_2^c	D_3^c
S_4	1	1	1	1	1	1	1	1	1
Z_2	—	—	—	—	—	—	—	—	—
	E_1^c	E_2^c	E_3^c	L_i	L_3	N_i^c	N_3^c	H_i^D	H_3^D
S_4	1	1	1'	2	1	2	1	2	1
Z_2	+	—	+	—	—	+	—	—	+
	H_i^U	H_3^U	S_i	S_3	G_a	G_a^c	Φ_i	Φ_3	Φ_a^c
S_4	2	1	2	1	3	3	2	1	3
Z_2	—	+	—	+	+	+	+	+	+

Table 2: $S_4 \times Z_2$ assignment of superfields (Where the index i of the S_4 doublets runs $i = 1, 2$, and the index a of the S_4 triplets runs $a = 1, 2, 3$.)

2.1 Higgs sector and hidden sector

Under the flavor symmetry given in Table 2, the superpotential of Higgs sector is given by,

$$\begin{aligned} W_H &= \lambda_1 S_3 (H_1^U H_1^D + H_2^U H_2^D) + \lambda_3 S_3 H_3^U H_3^D \\ &+ \lambda_4 H_3^U (S_1 H_1^D + S_2 H_2^D) + \lambda_5 (S_1 H_1^U + S_2 H_2^U) H_3^D. \end{aligned} \quad (3)$$

where one can take, without any loss of the generalities, $\lambda_{1,3,4,5}$ as real, by redefining four arbitrary fields of $\{S_i, S_3, H_i^U, H_3^U, H_i^D, H_3^D\}$. As only S_4 singlets H_3^U, H_3^D and S_3 couple to quarks and g-quarks respectively, they behave like MSSM Higgs and SM singlet respectively. Through the renormalization group equations, the squared masses of H_3^U, S_3 become negative, they develop VEVs and break gauge symmetry. As the result, the A-term $A_3 S_3 H_3^U H_3^D$ enforces H_3^D developing VEV. However, S_4 doublets do not develop VEVs. To generate the VEVs of S_4 doublets, we must add flavor breaking squared mass terms. The origin of flavor breaking terms is discussed below. Note that there is accidental $O(2)$ symmetry induced by the common rotation of the S_4 doublets in W_H .

As it is thought that the scalar squared masses are induced as the result of SUSY breaking in hidden sector, we assume flavor symmetry is broken at the same time. We assume hidden sector is described by flavor symmetric extension of O’Raifeartaigh model [7]. We introduce gauge singlet A, B_+, B_i, C_+, C_i and assign Z_2' charges to them to separate hidden sector from observable sector. We assume $U(1)_R$ symmetry is hold at the limit of infinite Planck scale, $M_P \rightarrow \infty$. The representations of hidden sector superfields under the $S_4 \times Z_2 \times Z_2' \times U(1)_R$ symmetry are given in table 3.

Under the symmetry given in Table 3, the superpotential of hidden sector is given by,

$$W_{\text{hidden}} = -M^2 A + m_+ B_+ C_+ + m(B_1 C_1 + B_2 C_2) + \frac{1}{2} \lambda_+ A C_+^2 + \frac{1}{2} \lambda A (C_1^2 + C_2^2). \quad (4)$$

	A	B_+	B_i	C_+	C_i
S_4	1	1	2	1	2
$Z_{2(4)}$	$+(0)$	$+(0)$	$-(1/2)$	$+(0)$	$-(1/2)$
Z'_2	$+$	$-$	$-$	$-$	$-$
$U(1)_R$	2	2	2	0	0

Table 3: $S_4 \times Z_{2(4)} \times Z'_2 \times U(1)_R$ assignment of superfields (Where the index i of the S_4 doublets runs $i = 1, 2$.)

For the F-terms of hidden sector superfields,

$$F_A = -M^2 + \frac{1}{2}\lambda_+ C_+^2 + \frac{1}{2}\lambda(C_1^2 + C_2^2), \quad (5)$$

$$F_{B_+} = m_+ C_+, \quad (6)$$

$$F_{B_1} = m C_1, \quad (7)$$

$$F_{B_2} = m C_2, \quad (8)$$

$$F_{C_+} = m_+ B_+ + \lambda_+ A C_+, \quad (9)$$

$$F_{C_1} = m B_1 + \lambda A C_1, \quad (10)$$

$$F_{C_2} = m B_2 + \lambda A C_2, \quad (11)$$

SUSY is spontaneously broken since it is impossible to satisfy the equations $F_A = 0$ and $F_{B_+} = F_{B_i} = 0$ at the same time. Notice that the flavor symmetry $S_4 \times Z_2$ is also broken spontaneously. As the superpotential has accidental $O(2)$ symmetry, the direction of S_4 doublet F-term defined by,

$$F_{B_1} = F_{C_B}, \quad F_{B_2} = F_{C_B}, \quad (12)$$

is described by free parameter θ_B . Although the spontaneous breaking of $O(2)$ results the appearance of Nambu-Goldstone boson (NGB), we assume this NGB does not cause any problem because the interactions of the NGB with observable sector particles are suppressed by Planck scale.

The SUSY breaking in hidden sector is mediated to observable sector by gravity through the non-renormalizable terms in Kähler potential as given by,

$$K \supset \frac{1}{M_P^2} [a_H (H_1 B_1 + H_2 B_2) (H_1 B_1 + H_2 B_2)^\dagger + (b_H B_+ H_3 (H_1 B_1 + H_2 B_2)^\dagger + h.c.)], \quad (13)$$

where $H = H^U, H^D, S$, and flavor symmetric terms and the contributions to other superfields are omitted. These terms induce scalar squared mass terms as follows

$$V_{FB} = m_{BH1}^2 |c_B H_1 + s_B H_2|^2 + [m_{BH2}^2 H_3 (c_B H_1 + s_B H_2)^\dagger + h.c.]. \quad (14)$$

If we substitute the VEV $\langle H_1 \rangle = v c_H, \langle H_2 \rangle = v s_H, \langle H_3 \rangle = v'$ for H_a , then we get

$$\begin{aligned} V_{FB} &= m_{BH1}^2 v^2 \cos^2(\theta_B - \theta_H) + [m_{BH2}^2 v v' \cos(\theta_B - \theta_H) + h.c.] \\ &= a [\cos(\theta_B - \theta_H) - b]^2 + \text{const.} \end{aligned} \quad (15)$$

At second line, we simplified the equation, because we are interested only in direction θ_H . As we can change the sign of b by the redefinition of the sign of v , we can define $b > 0$ without loss of generality. The minimum of potential V_{FB} is classified as follows

$$a > 0, b > 1 \quad : \quad \theta_H = \theta_B, \quad (16)$$

$$a > 0, b < 1 \quad : \quad \theta_H = \theta_B - \arccos b, \quad (17)$$

$$a < 0 \quad : \quad \theta_H = \theta_B + \pi, \quad (18)$$

from which one can see that the angle θ_H is controlled by free parameters a and b for the given θ_B . In this section, we assume the condition Eq.(16) is satisfied for H^U, H^D, S and select the common VEV direction θ_B . In this direction, the flavor symmetric part of Higgs potential has accidental $O(2)$ symmetry and do not depend on θ_H and θ_H that are fixed by V_{FB} . Therefore we can not assume V_{FB} as perturbation, even if flavor breaking parameter is small.

2.2 Flavon sector

The superpotential of flavon sector is given by,

$$\begin{aligned}
W_\Phi &= \frac{Y_1^\Phi}{2M_P} \Phi_3^2 [(\Phi_1^c)^2 + (\Phi_2^c)^2 + (\Phi_3^c)^2] \\
&+ \frac{Y_2^\Phi}{2M_P} (\Phi_1^2 + \Phi_2^2) [(\Phi_1^c)^2 + (\Phi_2^c)^2 + (\Phi_3^c)^2] \\
&+ \frac{Y_3^\Phi}{2M_P} \left\{ 2\sqrt{3}\Phi_1\Phi_2 [(\Phi_2^c)^2 - (\Phi_3^c)^2] + (\Phi_1^2 - \Phi_2^2) [(\Phi_2^c)^2 + (\Phi_3^c)^2 - 2(\Phi_1^c)^2] \right\} \\
&+ \frac{Y_4^\Phi}{2M_P} \Phi_3 \left\{ \sqrt{3}\Phi_1 [(\Phi_2^c)^2 - (\Phi_3^c)^2] + \Phi_2 [(\Phi_2^c)^2 + (\Phi_3^c)^2 - 2(\Phi_1^c)^2] \right\}, \tag{19}
\end{aligned}$$

and the flavor symmetric part of potential is given by,

$$\begin{aligned}
V &= -m_1^2|\Phi_3|^2 + m_2^2[|\Phi_1|^2 + |\Phi_2|^2] + m_3^2[|\Phi_1^c|^2 + |\Phi_2^c|^2 + |\Phi_3^c|^2] \\
&- \frac{A_1^\Phi}{2M_P} \Phi_3^2 [(\Phi_1^c)^2 + (\Phi_2^c)^2 + (\Phi_3^c)^2] \\
&- \frac{A_2^\Phi}{2M_P} (\Phi_1^2 + \Phi_2^2) [(\Phi_1^c)^2 + (\Phi_2^c)^2 + (\Phi_3^c)^2] \\
&- \frac{A_3^\Phi}{2M_P} \left\{ 2\sqrt{3}\Phi_1\Phi_2 [(\Phi_2^c)^2 - (\Phi_3^c)^2] + (\Phi_1^2 - \Phi_2^2) [(\Phi_2^c)^2 + (\Phi_3^c)^2 - 2(\Phi_1^c)^2] \right\} \\
&- \frac{A_4^\Phi}{2M_P} \Phi_3 \left\{ \sqrt{3}\Phi_1 [(\Phi_2^c)^2 - (\Phi_3^c)^2] + \Phi_2 [(\Phi_2^c)^2 + (\Phi_3^c)^2 - 2(\Phi_1^c)^2] \right\} \\
&+ \text{F-term} + \text{D-term}. \tag{20}
\end{aligned}$$

As this potential does not have accidental continuum symmetry, flavor breaking terms can be assumed as perturbation. We assume negative squared mass of Φ_3 pulls the trigger of $U(1)_Z$ breaking and Φ_3 and Φ_a^c develop VEVs along the D-flat direction. For the VEV of Φ_i , there are two possibilities in the flavor symmetry breaking. The one of them is S_3 symmetric vacuum defined as,

$$\Phi_3 = V, \quad \Phi_i = 0, \quad \Phi_a^c = \frac{V}{\sqrt{3}}(1, 1, 1), \tag{21}$$

and the other is S_3 breaking vacuum defined as,

$$\Phi_3 = Vc, \quad \Phi_i = Vs(0, 1), \quad \Phi_a^c = \frac{V}{\sqrt{2}}(0, 1, 1). \tag{22}$$

Another degenerated vacuums are given by acting S_4 translations on these VEVs, respectively. Note that spontaneous breaking of discrete symmetry causes domain wall problem. We assume flavor symmetry is not recovered in reheating era after inflation [8]. As the vacuum defined in Eq.(22) is not phenomenologically allowed because g-quark g_1, g_1^c become stable, we select the vacuum defined in Eq.(21) by tuning m_2^2 large enough.

From the simplified potential

$$V \sim -m_{SUSY}^2 |\Phi|^2 + \frac{|Y^\Phi|^2}{M_P^2} |\Phi^3|^2, \tag{23}$$

the size of flavon VEV is estimated as follows

$$\Phi \sim \sqrt{\frac{m_{SUSY} M_P}{Y^\Phi}}. \tag{24}$$

If we put $Y^\Phi \sim 0.01$ and $m_{SUSY} \sim 10$ TeV, then we get $V \sim 10^{12}$ GeV that is a favorable value to satisfy the constraint for g-quark and proton life time at the same time [9].

2.3 Quark and Lepton sector

As the flavor symmetry reduces the number of free parameters drastically, we can decide the value of free parameters by very few assumptions. For the quark sector, superpotential is given by

$$W_Q = Y_{ab}^U H_3^U Q_a U_b^c + Y_{ab}^D H_3^D Q_a D_b^c. \quad (25)$$

As the extra Higgs H_i^U, H_i^D do not couple to quarks, Higgs mediated flavor changing neutral currents are not induced.

For the lepton sector, superpotential is given by

$$\begin{aligned} W_L = & Y_2^N [H_1^U (L_1 N_2^c + L_2 N_1^c) + H_2^U (L_1 N_1^c - L_2 N_2^c)] \\ & + Y_3^N H_3^U L_3 N_3^c + Y_4^N L_3 (H_1^U N_1^c + H_2^U N_2^c) \\ & + Y_1^E E_1^c (H_1^D L_1 + H_2^D L_2) + Y_2^E E_2^c H_3^D L_3 + Y_3^E E_3^c (H_1^D L_2 - H_2^D L_1) \\ & + \frac{1}{2} Y_1^M \Phi_3 (N_1^c N_1^c + N_2^c N_2^c) + \frac{1}{2} Y_3^M \Phi_3 N_3^c N_3^c \\ & + \frac{1}{2} Y_2^M [\Phi_1 (2N_1^c N_2^c) + \Phi_2 (N_1^c N_1^c - N_2^c N_2^c)]. \end{aligned} \quad (26)$$

Without any loss of generalities, by the field redefinition, we can define $Y_{1,2,3}^E, Y_{1,3}^M, Y_{2,4}^N$ are real and non-negative and Y_2^M, Y_3^N are complex. We tune the angle $\theta_B = \theta_{23} = \frac{\pi}{4}$ and define the VEVs of scalar fields as follows

$$\begin{aligned} \langle H_1^U \rangle = \langle H_2^U \rangle &= \frac{1}{\sqrt{2}} v_u, \quad \langle H_3^U \rangle = v'_u, \quad \langle H_1^D \rangle = \langle H_2^D \rangle = \frac{1}{\sqrt{2}} v_d, \quad \langle H_3^D \rangle = v'_d, \\ \langle S_1 \rangle = \langle S_2 \rangle &= \frac{1}{\sqrt{2}} v_s, \quad \langle S_3 \rangle = v'_s, \\ \langle \Phi_1 \rangle = \langle \Phi_2 \rangle &= 0, \quad \langle \Phi_3 \rangle = V, \quad \langle \Phi_1^c \rangle = \langle \Phi_2^c \rangle = \langle \Phi_3^c \rangle = \frac{V}{\sqrt{3}}, \end{aligned} \quad (27)$$

and define the mass parameters as follows [10]

$$\begin{aligned} M_1 &= Y_1^M V, \quad M_3 = Y_3^M V, \\ m_2^\nu &= Y_2^N v_u, \quad m_3^\nu = |Y_3^N| v'_u, \quad m_4^\nu = Y_4^N v_u, \\ m_1^l &= Y_1^E v_d, \quad m_2^l = Y_2^E v'_d, \quad m_3^l = Y_3^E v_d. \end{aligned} \quad (28)$$

With these parameters, the mass matrices are given by

$$\begin{aligned} M_l &= \frac{1}{\sqrt{2}} \begin{pmatrix} m_1^l & 0 & -m_3^l \\ m_1^l & 0 & m_3^l \\ 0 & \sqrt{2} m_2^l & 0 \end{pmatrix}, \quad M_D = \frac{1}{\sqrt{2}} \begin{pmatrix} m_2^\nu & m_2^\nu & 0 \\ m_2^\nu & -m_2^\nu & 0 \\ m_4^\nu & m_4^\nu & \sqrt{2} e^{i\delta} m_3^\nu \end{pmatrix}, \\ M_R &= \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_3 \end{pmatrix}. \end{aligned} \quad (29)$$

Due to the seesaw mechanism, the neutrino mass matrix is given by

$$M_\nu = M_D M_R^{-1} M_D^t = \begin{pmatrix} \rho_2^2 & 0 & \rho_2 \rho_4 \\ 0 & \rho_2^2 & 0 \\ \rho_2 \rho_4 & 0 & \rho_4^2 + e^{2i\delta} \rho_3^2 \end{pmatrix}, \quad (30)$$

where

$$\rho_2 = \frac{m_2^\nu}{\sqrt{M_1}}, \quad \rho_4 = \frac{m_4^\nu}{\sqrt{M_1}}, \quad \rho_3 = \frac{m_3^\nu}{\sqrt{M_3}}. \quad (31)$$

The charged lepton mass matrix is diagonalized as follows

$$V_l^\dagger M_l^* M_l^t V_l = \text{diag}(m_e^2, m_\mu^2, m_\tau^2) = ((m_e^l)^2, (m_\mu^l)^2, (m_\tau^l)^2), \quad (32)$$

$$V_l = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & 1 \\ -\sqrt{2} & 0 & 0 \end{pmatrix}, \quad (33)$$

and those of the light neutrinos are given by

$$V_\nu^t M_\nu V_\nu = \text{diag}(e^{i(\phi_1-\phi)} m_{\nu_1}, e^{i(\phi_2+\phi)} m_{\nu_2}, m_{\nu_3}), \quad (34)$$

$$V_\nu = \begin{pmatrix} -\sin \theta_\nu & e^{i\phi} \cos \theta_\nu & 0 \\ 0 & 0 & 1 \\ e^{-i\phi} \cos \theta_\nu & \sin \theta_\nu & 0 \end{pmatrix}. \quad (35)$$

From the above equations, the MNS matrix is given by

$$V_{MNS} = V_l^\dagger V_\nu P_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2} e^{-i\phi} \cos \theta_\nu & -\sqrt{2} \sin \theta_\nu & 0 \\ \sin \theta_\nu & -e^{i\phi} \cos \theta_\nu & 1 \\ -\sin \theta_\nu & e^{i\phi} \cos \theta_\nu & 1 \end{pmatrix} P_\nu, \quad (36)$$

where

$$P_\nu = \text{diag}(e^{-i(\phi_1-\phi)/2}, e^{-i(\phi_2+\phi)/2}, 1). \quad (37)$$

As the value of θ_B is correctly tuned, experimental value of θ_{23} is realized. From the experimental constraints [11],

$$\tan \theta_\nu = \frac{1}{\sqrt{2}}, \quad m_{\nu_2}^2 - m_{\nu_1}^2 = 7.6 \times 10^{-5} (\text{eV}^2), \quad m_{\nu_2}^2 - m_{\nu_3}^2 = 2.5 \times 10^{-3} (\text{eV}^2), \quad (38)$$

the phase ϕ is constrained by the condition

$$r \cos \phi = 0.361, \quad r = \frac{\rho_2}{\rho_4}. \quad (39)$$

If we put the size of VEVs of Higgs fields as follows

$$v_u = 10, \quad v'_u = 155.3, \quad v_d = 2.0, \quad v'_d = 77.8 \quad (\text{GeV}), \quad (40)$$

we can fix Yukawa coupling constants as follows

$$Y_1^E = 0.875, \quad Y_3^E = 5.15 \times 10^{-2}, \quad Y_2^E = 6.25 \times 10^{-6}, \quad (41)$$

where we use the charged lepton masses [12] as follows

$$m_1^l = 1.75 \text{ GeV}, \quad m_2^l = 487 \text{ keV}, \quad m_3^l = 103 \text{ MeV}. \quad (42)$$

For the neutrinos, if we put

$$V = 10^{12} \text{ GeV}, \quad Y_1^M = Y_3^M = 1, \quad (43)$$

and assume $\phi = 0$, all Yukawa coupling constants are determined as follows

$$\begin{aligned} \phi = \phi_2 = 0, \quad \phi_1 = \pi, \quad \delta = \frac{\pi}{2}, \\ \rho_2^2 = 1.795 \times 10^{-2} \text{ eV}, \quad \rho_3^2 = 15.51 \times 10^{-2} \text{ eV}, \quad \rho_4^2 = 13.79 \times 10^{-2} \text{ eV}, \\ m_{\nu_1} = 5.240 \times 10^{-2} \text{ eV}, \quad m_{\nu_2} = 5.312 \times 10^{-2} \text{ eV}, \quad m_{\nu_3} = 1.795 \times 10^{-2} \text{ eV}, \\ m_2^\nu = 4.24 \text{ GeV}, \quad m_3^\nu = 12.45 \text{ GeV}, \quad m_4^\nu = 11.74 \text{ GeV}, \\ Y_2^N = 0.424, \quad Y_3^N = 0.080, \quad Y_4^N = 1.17. \end{aligned} \quad (44)$$

For the lepton sector, there is no flavor changing process as same as quark sector discussed above. Considering the interactions

$$\begin{aligned} \mathcal{L}_l = & Y_1^E \tau^c \left[l_\mu \left(\frac{H_2^D - H_1^D}{\sqrt{2}} \right) + l_\tau \left(\frac{H_1^D + H_2^D}{\sqrt{2}} \right) \right] - Y_2^E H_3^D e^c l_e \\ & + Y_3^E \mu^c \left[l_\mu \left(\frac{H_1^D + H_2^D}{\sqrt{2}} \right) + l_\tau \left(\frac{H_1^D - H_2^D}{\sqrt{2}} \right) \right], \end{aligned} \quad (45)$$

$\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ processes are forbidden because e does not couple to μ and τ . Furthermore, due to the unbroken S_2 symmetry such as $l_\mu \rightarrow -l_\mu, \mu^c \rightarrow -\mu^c, (H_1^D, H_2^D) \rightarrow (H_2^D, H_1^D), \tau \rightarrow \mu\gamma$ process is also forbidden.

2.4 g-quark sector

As the masses of g-quarks are given by

$$W_G = kS_3(G_1G_1^c + G_2G_2^c + G_3G_3^c), \quad (46)$$

the g-quark mass matrix is proportional to unit matrix. For the scalar g-quarks, as the contribution from soft flavor breaking term should be added, the degeneracy of masses may be broken. However, such effects can be assumed as perturbation. It is thought that dark matter does not have strong interaction, g-quark should not be stable if the reheating temperature is higher than g-quark mass. Under the symmetry defined in Table 1 and Table 2, g-quarks can decay through the non-renormalizable terms as follows

$$\begin{aligned} W_B = & \frac{1}{M_P^2} Y_{ab}^{QQG} \Phi_3 [G_1 \Phi_1^c + G_2 \Phi_2^c + G_3 \Phi_3^c] Q_a Q_b \\ & + \frac{1}{M_P^2} Y_{ab}^{UDG} \Phi_3 [G_1^c \Phi_1^c + G_2^c \Phi_2^c + G_3^c \Phi_3^c] U_a^c D_b^c \\ & + \frac{1}{M_P^2} Y_a^{EUG} \Phi_3 [G_1 \Phi_1^c + G_2 \Phi_2^c + G_3 \Phi_3^c] E_2^c U_a^c \\ & + \frac{1}{M_P^2} Y_a^{LsQG} \Phi_3 [G_1^c \Phi_1^c + G_2^c \Phi_2^c + G_3^c \Phi_3^c] L_3 Q_a \\ & + \frac{1}{M_P^2} Y_a^{LaQG} \Phi_3 [\sqrt{3} L_1 (G_2^c \Phi_2^c - G_3^c \Phi_3^c) + L_2 (G_2^c \Phi_2^c + G_3^c \Phi_3^c - 2G_1^c \Phi_1^c)] Q_a. \end{aligned} \quad (47)$$

Here we estimate the g-quark life time by the Y^{EUG} interaction. If we put

$$Y_1^{EUG} = Y_2^{EUG} = Y_3^{EUG} = 1, \quad (48)$$

then the interaction is given by

$$\mathcal{L}_g = \frac{(A_{RF}^{EUG})_S V^2}{\sqrt{3} M_P^2} E_2^c g_1 (u_1^c + u_2^c + u_3^c), \quad (49)$$

where E_2^c is right handed selectron. The renormalization factor $(A_{RF}^{EUG})_S$ is evaluated by the RGE

$$(4\pi) \frac{d \ln Y_a^{EUG}}{d \ln \mu} = -\frac{16}{3} \alpha_s, \quad (50)$$

and given by

$$(A_{RF}^{EUG})_S = \left(\frac{M_P}{M_Z} \right)^{4\alpha_s/3\pi} = \left(\frac{2.43 \times 10^{18}}{91} \right)^{0.05008} = 6.647, \quad (51)$$

where only QCD correction is accounted. This approximation is not bad because the beta function of the coupling constant of strong interaction g_s vanishes at 1-loop level in our model, which makes the contribution of α_s dominant in the RGE of Y^{EUG} .

Using the interaction Eq.(49), we calculate the g-quark decay width. For simplicity, we assume u_a^c in Eq.(49) are mass eigenstates. Requiring the life time of g-quark is shorter than 0.1 sec (otherwise the success of BBN is spoiled [13]) as follows

$$\Gamma(g_1) = 3 \left(\frac{(A_{RF}^{EUG})_S V^2}{3M_P^2} \right)^2 \frac{M_g}{16\pi} > \frac{1}{0.1 \text{ sec}}, \quad (52)$$

we get

$$\frac{M_g}{1000 \text{ GeV}} \left(\frac{V}{M_P} \right)^4 > 2.25 \times 10^{-26}, \quad (53)$$

which bounds the VEV size of flavon from below.

Finally, using the interaction $Y^{QQG} - Y^{EUG}$, we estimate the proton decay width. Integrating out the scalar g-quarks, we get the effective four-Fermi interactions as follows

$$\mathcal{L}_{p \rightarrow e^+ \pi^0} = \frac{V^4}{M_P^4 M_G^2} Y_{ab}^{QQG} Y_c^{EUG} A_{RF} \bar{q}_a \bar{q}_b u_c^c e^c, \quad (54)$$

where

$$A_{RF} = (A_{RF}^{EUG})_S (A_{RF}^{QQG})_S (A_{RF})_L \quad [14]. \quad (55)$$

The renormalization factor $(A_{RF}^{QQG})_S$ is evaluated by the REG

$$(4\pi) \frac{d \ln Y_a^{QQG}}{d \ln \mu} = -\frac{24}{3} \alpha_s, \quad (56)$$

and given by

$$(A_{RF}^{QQG})_S = \left(\frac{M_P}{M_Z} \right)^{2\alpha_s/\pi} = \left(\frac{2.43 \times 10^{18}}{91} \right)^{0.07512} = 17.139. \quad (57)$$

As the long distant part of renormalization factor is given by

$$(A_{RF})_L = \left(\frac{\alpha_s(1\text{GeV})}{\alpha_s(m_b)} \right)^{6/25} \left(\frac{\alpha_s(m_b)}{\alpha_s(M_Z)} \right)^{6/23} = 1.4 \quad [15], \quad (58)$$

we get

$$A_{RF} = 159.5. \quad (59)$$

In the quark mass basis, Eq.(54) is rewritten as follows

$$\mathcal{L}_{p \rightarrow e^+ \pi^0} = \frac{V^4}{M_P^4 M_G^2} [2(L_u^T Y^{UDG} L_u)_{ab} (Y^{EUG} R_u)_c] A_{RF} \bar{u}'_a \bar{d}'_b (u_c^c)' e^c, \quad (60)$$

$$\bar{u} = L_u \bar{u}', \quad \bar{d} = L_d \bar{d}', \quad u^c = R_u (u^c)'. \quad (61)$$

For simplicity, we put

$$[2(L_u^T Y^{UDG} L_u)_{11} (Y^{EUG} R_u)_1] = 1, \quad (62)$$

then the proton decay width is given by

$$\Gamma(p \rightarrow \pi^0 + e^+) = \frac{m_p}{64\pi f_\pi^2} \left[\left(\frac{V}{M_P} \right)^4 \frac{A_{RF}^2}{M_G^2} \right]^2 (1 + F + D)^2 \left(1 - \frac{m_{\pi^0}^2}{m_p^2} \right)^2 \alpha_p^2 \quad [16]. \quad (63)$$

If we put

$$\begin{aligned} F &= 0.47, \quad D = 0.80, \quad \alpha_p = -0.012 \text{ GeV}^3, \quad f_\pi = 130 \text{ MeV}, \quad [17] \\ m_{\pi^0} &= 135 \text{ MeV}, \quad m_p = 940 \text{ MeV}, \quad [11] \end{aligned} \quad (64)$$

then we get

$$\Gamma(p \rightarrow \pi^0 + e^+) = (5.01 \times 10^{-12} \text{ GeV}) \left[\left(\frac{V}{M_P} \right)^4 \frac{(1000 \text{ GeV})^2}{M_G^2} \right]^2. \quad (65)$$

From the experimental bound $\tau(p \rightarrow \pi^0 + e^+) > 1600 \times 10^{30} [\text{years}]$ [11], the VEV size of flavon is bounded from above as follows

$$\left[\left(\frac{V}{M_P} \right)^4 \left(\frac{1000 \text{ GeV}}{M_G} \right)^2 \right]^2 < 2.60 \times 10^{-54}. \quad (66)$$

Hereafter we assume the approximation $M_g = M_G$ is held for simplicity. From Eq.(53) and Eq.(66), the allowed region for V is given by (see Fig.1)

$$2.25 \times 10^{-26} \left(\frac{1000 \text{ GeV}}{M_G} \right) < \left(\frac{V}{M_P} \right)^4 < 1.61 \times 10^{-27} \left(\frac{M_G}{1000 \text{ GeV}} \right)^2. \quad (67)$$

This inequality holds when the mass bound,

$$M_G > 2.41 \text{ TeV}, \quad (68)$$

is satisfied.

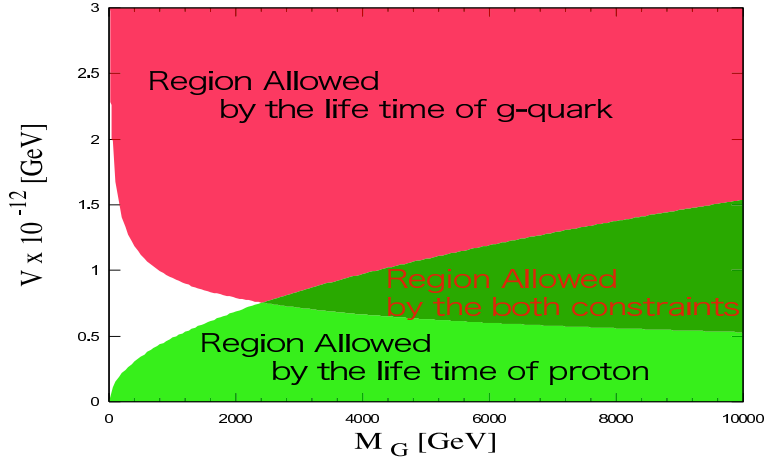


Figure 1: M_G versus V : The pink region comes from the constraint of the life time of the g-quark, which should be less than 0.1 sec. The green region comes from the constraint of the proton stability. The black region is allowed by the both constraints. The heavier of M_G , the wider the allowed region is.

Before ending this section, we discuss the unsatisfactory point of this model. Considering the mass spectrum of the quarks and charged leptons, it is expected that the trilinear coupling of first generation is multi suppressed by the suppression mechanism of Yukawa couplings and S_4 symmetry. If it is true, as the proton decay width accommodates another suppression factor, the experimental verification of proton decay seems to be impossible. Therefore the suppression by the gauge non-singlet flavon may be too strong. To improve this point, we modify the flavon sector in next section.

3 $S_4 \times Z_4 \times Z_9$ flavor symmetric model

In this section we introduce Froggatt-Nielsen mechanism to explain Yukawa hierarchy [18], and the flavon sector is modified as follows. To realize $O(10^2)$ hierarchy, we introduce Z_9 symmetry and add gauge and S_4 singlet X as Froggatt-Nielsen (FN) flavon. To weaken the over suppression of trilinear terms, we replace the flavon Φ_a^c by S_4 singlet Φ^c and gauge singlet T which is assigned to S_4 triplet. To forbid renormalizable terms of T , the Z_2 symmetry is replaced by Z_4 . The flavor representations of superfields are given in Table 4.

3.1 Flavon sector

The leading terms of flavons are given as follows

$$W_F = W_T + W_\Phi + W_X, \quad (69)$$

$$W_T = \frac{1}{M_P} \left[\frac{1}{4} Y_1^T (T_1^4 + T_2^4 + T_3^4) + \frac{1}{2} Y_2^T (T_1^2 T_2^2 + T_1^2 T_3^2 + T_2^2 T_3^2) \right], \quad (70)$$

	Q_1	Q_2	Q_3	U_1^c	U_2^c	U_3^c	D_1^c	D_2^c	D_3^c	E_1^c	E_2^c	E_3^c	L_i	L_3
S_4	1	1	1	1	1	1	1	1	1	1	1	1'	2	1
Z_4	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	0	1/2	0	1/2	1/2
Z_9	1/9	1/9	0	2/9	1/9	0	2/9	1/9	1/9	0	2/9	1/9	0	1/9
	N_i^c	N_3^c	H_i^U	H_3^U	H_i^D	H_3^D	S_i	S_3	G_a	G_a^c	T_a	Φ	Φ^c	X
S_4	2	1	2	1	2	1	2	1	3	3	3	1	1'	1
Z_4	0	1/2	1/2	0	1/2	0	1/2	0	1/4	3/4	1/4	0	1/2	0
Z_9	0	0	0	0	0	0	0	0	0	0	0	0	0	8/9

Table 4: $S_4 \times Z_4 \times Z_9$ assignment of superfields (Where the index i of the S_4 doublets runs $i = 1, 2$, and the index a of the S_4 triplets runs $a = 1, 2, 3$.)

$$W_X = \frac{1}{6M_P^6} X^9, \quad (71)$$

$$W_\Phi = \frac{1}{2M_P} Y^\Phi (\Phi \Phi^c)^2. \quad (72)$$

The VEV size of gauge non-singlet is estimated by Eq.(24). Now we change the value of $V = \langle \Phi \rangle$ to 10^{11} GeV which is given by naive estimation when we put $Y^\Phi \sim 1$. This affects neutrino Yukawa couplings given in Eq.(44) as follows

$$Y_2^N \rightarrow \frac{Y_2^N}{\sqrt{10}} = 0.134, \quad Y_3^N \rightarrow \frac{Y_3^N}{\sqrt{10}} = 0.025, \quad Y_4^N \rightarrow \frac{Y_4^N}{\sqrt{10}} = 0.370. \quad (73)$$

The VEV size of FN flavon is estimated as follows

$$\epsilon = \left(\frac{\langle X \rangle}{M_P} \right) \sim \left(\frac{m_{SUSY}}{M_P} \right)^{\frac{1}{7}} \sim 10^{-2}. \quad (74)$$

S_4 symmetric part of potential of T is given by

$$\begin{aligned}
V_T = & -m^2(|T_1|^2 + |T_2|^2 + |T_3|^2) \\
& - \frac{1}{M_P} \left[\frac{1}{4} B_1^T (T_1^4 + T_2^4 + T_3^4) + \frac{1}{2} B_2^T (T_1^2 T_2^2 + T_1^2 T_3^2 + T_2^2 T_3^2) \right] \\
& + \frac{1}{M_P^2} [|Y_1^T T_1^3 + Y_2^T T_1 (T_2^2 + T_3^2)|^2 + |Y_1^T T_2^3 + Y_2^T T_2 (T_1^2 + T_3^2)|^2 \\
& + |Y_1^T T_3^3 + Y_2^T T_3 (T_1^2 + T_2^2)|^2], \quad (75)
\end{aligned}$$

which has minimum in the VEV direction given by

$$T_a = \frac{V_T}{\sqrt{3}} (1, 1, 1). \quad (76)$$

As same as the gauge non-singlet model, we can assume flavor breaking term as perturbation. As the size of V_T is at the same order as V , we put

$$\frac{V_T}{M_P} = 10^{-8}. \quad (77)$$

3.2 Quark and Lepton sector

Due to the Z_9 symmetry, the effective Yukawa coupling constants accommodate power of ϵ through the superpotential

$$W_Q = \sum_{ij} \left(\frac{X}{M_P} \right)^{9(q_i + u_j)} (Y_{ij}^U)_0 H_3^U Q_i U_j^c + \sum_{ij} \left(\frac{X}{M_P} \right)^{9(q_i + d_j)} (Y_{ij}^D)_0 H_3^D Q_i D_j^c, \quad (78)$$

where q_i, u_i, d_i are Z_9 charge of Q_i, U_i^c, D_i^c respectively. As the results, the mass matrices of quarks are given as follows

$$M_u \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \text{diag}(\epsilon^3, \epsilon^2, 1) \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} v'_u = L_u \text{diag}(m_{u,c,t}) R_u^\dagger, \quad (79)$$

$$M_d \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \text{diag}(\epsilon^3, \epsilon^2, \epsilon) \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} v'_d = L_d \text{diag}(m_{d,s,b}) R_d^\dagger, \quad (80)$$

from which we get Cabbibo-Kobayashi-Maskawa matrix

$$V_{CKM} = L_u^\dagger L_d \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}, \quad (81)$$

and quark masses divided by experimental values respectively,

$$\begin{aligned} \frac{m_u}{(m_u)_{\text{exp}}} &= \frac{(A_{RF}^Y)_S \epsilon^3 v'_u}{1.3 \times 10^{-3}} = 0.79, & \frac{m_c}{(m_c)_{\text{exp}}} &= \frac{(A_{RF}^Y)_S \epsilon^2 v'_u}{0.624} = 0.17, & \frac{m_t}{(m_t)_{\text{exp}}} &= \frac{v'_u}{173} = 0.90, \\ \frac{m_d}{(m_d)_{\text{exp}}} &= \frac{(A_{RF}^Y)_S \epsilon^3 v'_d}{2.9 \times 10^{-3}} = 0.18, & \frac{m_s}{(m_s)_{\text{exp}}} &= \frac{(A_{RF}^Y)_S \epsilon^2 v'_d}{0.055} = 0.94, & \frac{m_b}{(m_b)_{\text{exp}}} &= \frac{(A_{RF}^Y)_S \epsilon v'_d}{2.89} = 1.8, \end{aligned} \quad (82)$$

where $\epsilon = 0.01, v'_u = 155.3 \text{ GeV}, v'_d = 77.8 \text{ GeV}, (A_{RF}^Y)_S = (A_{RF}^{EUG})_S = 6.647$ are used. The renormalization factor of top-Yukawa coupling is neglected because it has infrared quasi-fixed point. For the lepton sector, the Yukawa coupling constants divided by required values given in Eq.(42) and Eq.(73) are given by

$$\begin{aligned} \frac{Y_1^E}{(Y_1^E)_{\text{exp}}} &= \frac{1}{0.875} = 1.1, & \frac{Y_3^E}{(Y_3^E)_{\text{exp}}} &= \frac{\epsilon}{5.15 \times 10^{-2}} = 0.19, & \frac{Y_2^E}{(Y_2^E)_{\text{exp}}} &= \frac{\epsilon^3}{6.25 \times 10^{-6}} = 0.16, \\ \frac{Y_2^N}{(Y_2^N)_{\text{exp}}} &= \frac{1}{0.134} = 7.5, & \frac{Y_3^N}{(Y_3^N)_{\text{exp}}} &= \frac{\epsilon}{0.025} = 0.40, & \frac{Y_4^N}{(Y_4^N)_{\text{exp}}} &= \frac{1}{0.370} = 2.7. \end{aligned} \quad (83)$$

Where we used running masses of quarks and charged leptons [12]:

$$\begin{aligned} m_u(m_Z) &= 1.28_{-0.39}^{+0.50} (\text{MeV}), & m_c(m_Z) &= 624 \pm 83 (\text{MeV}), & m_t(m_Z) &= 172.5 \pm 3.0 (\text{GeV}), \\ m_d(m_Z) &= 2.91_{-1.20}^{+1.24} (\text{MeV}), & m_s(m_Z) &= 55_{-15}^{+16} (\text{MeV}), & m_b(m_Z) &= 2.89 \pm 0.09 (\text{GeV}), \\ m_e(m_Z) &= 0.48657 (\text{MeV}), & m_\mu(m_Z) &= 102.72 (\text{MeV}), & m_\tau(m_Z) &= 1746 (\text{MeV}). \end{aligned} \quad (84)$$

The discrepancies between the estimated values and experimental values in Eq.(82) and Eq.(83) are easily recovered by multiplying $O(1)$ coefficients $(Y)_0$. CKM matrix

$$(V_{CKM})_{\text{exp}} \simeq \begin{pmatrix} 1 & 0.23 & 0.4 \times 10^{-2} \\ 0.23 & 1 & 4.1 \times 10^{-2} \\ 0.8 \times 10^{-2} & 3.9 \times 10^{-2} & 1 \end{pmatrix}, \quad [11] \quad (85)$$

is also recovered from Eq.(81). Therefore our procedure works well in quark and lepton sectors.

3.3 g-quark sector

The leading terms of single g-quark interactions are given by

$$\begin{aligned} W_B &= \frac{1}{M_P} \left(\frac{X}{M_P} \right)^{9(u_a+d_a)} (Y_{ab}^{UDG})_0 [T_1 G_1^c + T_2 G_2^c + T_3 G_3^c] U_a^c D_b^c \\ &+ \frac{1}{M_P} \left(\frac{X}{M_P} \right)^{9q_a+1} (Y_a^{QL_s G})_0 [T_1 G_1^c + T_2 G_2^c + T_3 G_3^c] Q_a L_3 \\ &+ \frac{1}{M_P} \left(\frac{X}{M_P} \right)^{9q_a} (Y_a^{QL_d G})_0 [\sqrt{3}(T_2 G_2^c - T_3 G_3^c) L_1 + (T_2 G_2^c + T_3 G_3^c - 2T_1 G_1^c) L_2] Q_a, \end{aligned} \quad (86)$$

where the contribution from $E_1^c \supset \tau^c$ is omitted because $p \rightarrow \tau^+ X$ is impossible. Note that Y^{QQG}, Y^{EUG} are suppressed by $(V_T/M_P)^3$. In the quark mass basis, trilinear coupling matrix and vectors are given as follows

$$(R_u^T Y^{UDG} R_d)_{ab} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \end{pmatrix} \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \end{pmatrix}, \quad (87)$$

$$(Y^{QL_s G} L_u)_a \sim \epsilon(\epsilon, \epsilon, 1) \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \sim (\epsilon^2, \epsilon^2, \epsilon), \quad (88)$$

$$(Y^{QL_d G} L_u)_a \sim (\epsilon, \epsilon, 1) \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \sim (\epsilon, \epsilon, 1). \quad (89)$$

As the coupling constants are large enough, the problem of long life time of g-quark is solved. Integrating out the scalar g-quarks, would-be the most largest contribution to proton decay is given by

$$\mathcal{L}_{p \rightarrow \mu^+ K^0} = \frac{\epsilon^4}{M_P^2 M_G^2} \langle \Phi_3 \rangle^2 (\bar{u}^c)' (\bar{s}^c)' u' [\sqrt{3} e_1 (\langle T_2 \rangle^2 - \langle T_3 \rangle^2) + e_2 (\langle T_2 \rangle^2 + \langle T_3 \rangle^2 - 2 \langle T_1 \rangle^2)], \quad (90)$$

where e_1, e_2 are linear combinations of μ and τ . Interestingly, this interaction vanishes in the VEV direction given in Eq.(76). This means the contributions from three scalar g-quarks are canceled. Therefore the dominant contribution to proton decay is given by

$$\mathcal{L}_{p \rightarrow e^+ K^0} = \frac{\epsilon^5 V_T^2}{M_P^2 M_G^2} A_{RF} (\bar{u}^c)' (\bar{s}^c)' u' e, \quad (91)$$

from which we get

$$\Gamma(p \rightarrow e^+ + K^0) = \frac{m_p}{32\pi f_\pi^2} \left[\epsilon^5 \left(\frac{V_T}{M_P} \right)^2 \frac{A_{RF}}{M_G^2} \right]^2 \left[-1 + \frac{m_N}{m_{B'}} (F - D) \right]^2 \left(1 - \frac{m_{K^0}^2}{m_p^2} \right)^2 \alpha_p^2. \quad (92)$$

Substituting

$$\epsilon = 0.01, \quad m_N = m_p = 940 \text{ MeV}, \quad m_{B'} = \frac{m_\Lambda + m_\Sigma}{2} = 1150 \text{ MeV}, \quad m_{K^0} = 498 \text{ MeV}, \quad [11] \quad (93)$$

and Eq.(59), Eq.(64) and Eq.(77), we get

$$\Gamma(p \rightarrow e^+ + K^0) = 1.69 \times 10^{-64} \left(\frac{1000 \text{ GeV}}{M_G} \right)^4 \text{ GeV}. \quad (94)$$

For the experimental bound $\tau(p \rightarrow e^+ + K^0) > 150 \times 10^{30} [\text{years}]$ [11], mass bound as follows

$$M_G > 1.0 \text{ TeV} \quad (95)$$

must be satisfied. The experimental bound $\tau(p \rightarrow \bar{\nu} + K^+) > 670 \times 10^{30} [\text{years}]$ [11] for the operator

$$\mathcal{L}_{p \rightarrow \bar{\nu} K^+} = \frac{\epsilon^5 V_T^2}{M_P^2 M_G^2} A_{RF} (\bar{u}^c)' (\bar{s}^c)' d' \nu, \quad (96)$$

gives weaker mass bound as follows

$$\begin{aligned} \Gamma(p \rightarrow \bar{\nu} + K^+) &= \frac{m_p}{32\pi f_\pi^2} \left[\epsilon^5 \left(\frac{V_T}{M_P} \right)^2 \frac{A_{RF}}{M_G^2} \right]^2 \left[\frac{2}{3} \frac{m_N}{m_{B'}} D \right]^2 \left(1 - \frac{m_{K^0}^2}{m_p^2} \right)^2 \alpha_p^2 \\ &= 0.20 \times 10^{-64} \left(\frac{1000 \text{ GeV}}{M_G} \right)^4 \text{ GeV}, \end{aligned} \quad (97)$$

$$M_G > 0.9 \text{ TeV}. \quad (98)$$

Note that these bounds should not be taken seriously, because there is $O(10)$ ambiguity coming from SUSY breaking parameter in V_T , this constraint also has such ambiguity. The important point is that we can expect the experimental observation of proton decay for TeV scale g-quark in near future.

Finally we give a short comment about flavor breaking effects on cancellation. Due to the perturbation from flavor breaking squared mass terms, scalar g-quark squared mass and squared VEVs of flavons receive $O(m_B^2/m_{SUSY}^2)$ contaminations, where m_B^2 is flavor breaking squared mass. As the results, the cancellation is spoiled. However, if flavor breaking terms are small enough to satisfy the condition

$$r_B = \frac{m_B^2}{m_{SUSY}^2} < \epsilon, \quad (99)$$

the cancellation mechanism works effectively. In the opposite case of $r_B > \epsilon$, $p \rightarrow \mu^+ K^0$ dominates the proton decay width. Therefore the size of r_B affects proton decay channels significantly. There is interesting correlation through r_B between proton decay channel and degree of degeneracy of scalar g-quark masses. Note that too small r_B causes the appearance of pseudo-Nambu-Goldstone boson (pNGB). However this may be not a serious problem, because if there is the VEV hierarchy such as $v_s \gg v_u, v_d$, which makes S_i dominant in pNGB, then the interactions of this pNGB with quarks, leptons and weak bosons are very weak.

4 Dirac neutrino model

As is shown in previous section, the stability of proton is realized by strong suppression factor of $\langle T \rangle / M_P \sim 10^{-8}$. It is not unnatural to expect that the small neutrino mass is realized by same mechanism. In this section, we construct Dirac neutrino model based on S_4 flavor symmetry. We eliminate Φ, Φ^c and change the flavor assignment as given in Table 5. To forbid the bilinear term $H_i^U L_i$, we change the extra gauge symmetry to one linear combination of two extra U(1) gauge symmetries as defined by

$$X_\theta = X \cos \theta + Z \sin \theta, \quad (100)$$

and assume RHN and S have non-zero charge of X_θ . The value of θ does not affect our analysis.

	Q_1	Q_2	Q_3	U_1^c	U_2^c	U_3^c	D_1^c	D_2^c	D_3^c
S_4	1	1	1	1	1	1	1	1	1
Z_4	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2
Z_9	1/9	1/9	0	2/9	1/9	0	2/9	1/9	1/9
	E_1^c	E_2^c	E_3^c	L_i	L_3	H_i^U	H_3^U	H_i^D	H_3^D
S_4	1	1	1'	2	1	2	1	2	1
Z_4	0	1/2	0	1/2	1/2	1/2	0	1/2	0
Z_9	0	3/9	1/9	0	0	0	0	0	0
	S_i	S_3	N_a^c	G_a	G_a^c	T_a	X		
S_4	2	1	3	3	3	3	1		
Z_4	1/2	0	3/4	1/4	3/4	1/4	0		
Z_9	0	0	2/9	0	0	0	8/9		

Table 5: $S_4 \times Z_4 \times Z_9$ assignment of superfields (Where the index i of the S_4 doublets runs $i = 1, 2$, and the index a of the S_4 triplets runs $a = 1, 2, 3$.)

As the quark and charged lepton Yukawa interactions are not modified, we consider only Yukawa interaction of neutrino which is given by

$$\begin{aligned}
W_N &= \frac{X^2}{M_P^3} (Y_1^N)_0 (T_1 N_1^c + T_2 N_2^c + T_3 N_3^c) (H_1^U L_1 + H_2^U L_2) \\
&- \frac{X^2}{M_P^3} (Y_2^N)_0 [\sqrt{3} (T_2 N_2^c - T_3 N_3^c) (H_1^U L_2 + H_2^U L_1) + (T_2 N_2^c + T_3 N_3^c - 2T_1 N_1^c) (H_1^U L_1 - H_2^U L_2)] \\
&- \frac{X^2}{M_P^3} (Y_3^N)_0 [\sqrt{3} (T_2 N_2^c - T_3 N_3^c) H_1^U + (T_2 N_2^c + T_3 N_3^c - 2T_1 N_1^c) H_2^U] L_3.
\end{aligned} \quad (101)$$

Substituting the VEVs given in Eq.(74), Eq.(76) and Eq.(77) for X and T_a , we get the effective superpotential as follows

$$\begin{aligned} W_N &= Y_1^N (H_1^U L_1 + H_2^U L_2) (N_1^c + N_2^c + N_3^c) \\ &+ Y_2^N [\sqrt{3}(N_3^c - N_2^c)(H_1^U L_2 + H_2^U L_1) + (2N_1^c - N_2^c - N_3^c)(H_1^U L_1 - H_2^U L_2)] \\ &+ Y_3^N L_3 [\sqrt{3}(N_3^c - N_2^c)H_1^U + (2N_1^c - N_2^c - N_3^c)H_2^U], \end{aligned} \quad (102)$$

$$Y_{1,2,3}^N = (Y_{1,2,3}^N)_0 \epsilon^2 \left(\frac{V_T}{\sqrt{3}M_P} \right) \sim O(10^{-12}), \quad (103)$$

from which Dirac neutrino mass matrix is given by

$$M_D = \begin{pmatrix} (m_1 + 2m_2)c_u & m_1c_u - m_2(c_u + \sqrt{3}s_u) & m_1c_u - m_2(c_u - \sqrt{3}s_u) \\ (m_1 - 2m_2)s_u & m_1s_u + m_2(s_u - \sqrt{3}c_u) & m_1s_u + m_2(s_u + \sqrt{3}c_u) \\ 2m_3s_u & -m_3(s_u + \sqrt{3}c_u) & m_3(-s_u + \sqrt{3}c_u) \end{pmatrix}, \quad (104)$$

$$m_1 = Y_1^N v_u, \quad m_2 = Y_2^N v_u, \quad m_3 = Y_3^N v_u, \quad (105)$$

where we can define m_1 and m_3 as real and non-negative and m_2 as complex without loss of generality. In the VEV direction $\theta_u = \theta_d = \theta_B$, two large angles of charged lepton and neutrino mixing matrix are canceled and make it difficult to realize two large mixing angles of MNS matrix. Therefore we select the condition Eq.(17) and put $\theta_d = 0$ and $\theta_u = \frac{\pi}{4}$, then the charged lepton mass matrix is given by

$$M_l = \begin{pmatrix} m_1^e & 0 & 0 \\ 0 & 0 & m_3^e \\ 0 & m_2^e & 0 \end{pmatrix}, \quad (106)$$

from which we get

$$V_l^T M_l M_l^T V_l = \text{diag}((m_2^e)^2, (m_3^e)^2, (m_1^e)^2) = \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \quad V_l = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (107)$$

To realize experimental results, the conditions given as follows

$$V_\nu^\dagger M_D^* M_D^T V_\nu = \text{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2) = M_{\text{diag}}^2, \quad (108)$$

$$V_\nu = V_l V_{MNS} = \frac{1}{\sqrt{6}} V_l \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -\lambda^* \\ 0 & 1 & 0 \\ \lambda & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 1 & 0 \\ -1 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{3} \end{pmatrix}, \quad (109)$$

$$M_D^* M_D^T = V_\nu M_{\text{diag}}^2 V_\nu^\dagger = \text{diag}(1, 1, 1) m_{\nu_2}^2 + \Delta m_{32}^2 V_\nu \text{diag}(-r_\nu, 0, 1) V_\nu^\dagger, \quad (110)$$

$$r_\nu = \frac{\Delta m_{21}^2}{\Delta m_{32}^2}, \quad (111)$$

must be satisfied. Eq.(110) is rewritten as follows

$$\begin{aligned} &\begin{pmatrix} (3/2)m_1^2 + 6|m_2|^2 - m_{\nu_2}^2 & (3/2)m_1^2 & 6m_2^* m_3 \\ (3/2)m_1^2 & (3/2)m_1^2 + 6|m_2|^2 - m_{\nu_2}^2 & 0 \\ 6m_2 m_3 & 0 & 6m_3^2 - m_{\nu_2}^2 \end{pmatrix} \\ &= \frac{\Delta m_{32}^2}{6} \begin{pmatrix} 3 - r_\nu & 3 + r_\nu & -2r_\nu - 3\sqrt{2}\lambda \\ 3 + r_\nu & 3 - r_\nu & 2r_\nu - 3\sqrt{2}\lambda \\ -2r_\nu - 3\sqrt{2}\lambda^* & 2r_\nu - 3\sqrt{2}\lambda^* & -4r_\nu \end{pmatrix}, \end{aligned} \quad (112)$$

where $O(\lambda^2)$ terms are neglected. From this equation, we get

$$\sin \theta_{13} = \lambda = \lambda^* = \frac{\sqrt{2}}{3} r_\nu = \frac{\sqrt{2}}{3} \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = 0.014 \quad (\theta_{13} = 0.8^\circ), \quad (113)$$

$$\begin{aligned} m_1 &= 0.029 \text{ eV}, \quad m_2 = -0.0033 \text{ eV}, \quad m_3 = 0.0025 \text{ eV} \\ m_{\nu_1} &= 0.0035 \text{ eV}, \quad m_{\nu_2} = 0.0094 \text{ eV}, \quad m_{\nu_3} = 0.051 \text{ eV}, \end{aligned} \quad (114)$$

$$Y_1^N = 2.9 \times 10^{-12}, \quad Y_2^N = -0.33 \times 10^{-12}, \quad Y_3^N = 0.25 \times 10^{-12}. \quad (115)$$

The small discrepancies between Eq.(103) and Eq.(115) are recovered by multiplying $O(1)$ coefficients $(Y^N)_0$.

By the modification of Z_9 charge of L_3 , the proton decay width is dominated by $p \rightarrow e^+ K^0$ given in Eq.(91) because suppression factor is reduced from ϵ^5 to ϵ^4 . From the experimental bound, we get

$$M_G > \frac{1.0 \text{ TeV}}{\sqrt{\epsilon}} = 10 \text{ TeV}. \quad (116)$$

Note that Y^{NDG} is suppressed by $(V_T/M_P)^2$.

5 Conclusion

We have considered proton stability based on S_4 symmetric extra $U(1)$ models. Without Froggatt-Nielsen mechanism, most stringent bound for proton decay channel is given by $\tau(p \rightarrow e^+ \pi^0)$. As the single quark interaction is doubly suppressed by the VEV of gauge non-singlet flavon, g-quark life time becomes very long. Therefore the allowed region for flavon VEV is very narrow for TeV scale g-quark.

Introducing Froggatt-Nielsen mechanism, as we can weaken the S_4 flavon VEV suppression, g-quark life time becomes short enough. From the naive power counting, we can expect $p \rightarrow \mu^+ K^0$ would dominate the proton decay width, however, corresponding operator vanishes by cancellation and do not contribute to proton decay. Therefore our model predicts $p \rightarrow e^+ K^0$ dominates the proton decay width. This conclusion is not modified in Dirac neutrino model.

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